# EENG 426/CPSC 459/ENAS 876
# Silicon Compilation

**Non-deterministic selections**

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## Arbitration

Consider the following CHP program:

$$
\begin{array}{l}
*[[\overline{X} \longrightarrow X?x \\
\;\; | \overline{Y} \longrightarrow Y?x \\
\;\; ]; \\
\;\; Z!x \\
]
\end{array}
$$

When $\overline{X}$ and $\overline{Y}$ are both **true**, we have to pick one of them and execute the appropriate branch of the selection statement.

Arbitration is the mechanism that picks one of two alternatives, deciding which alternative came "first."

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## Arbiters

An **arbiter** is the following process:

$$
\begin{array}{ll}
Arb(a,b,u,v) \equiv & *[[a \longrightarrow u\uparrow; [\neg a]; u\downarrow \\
& \;\; | b \longrightarrow v\uparrow; [\neg b]; v\downarrow \\
& ]]
\end{array}
$$

The process does a handshake on $(a,u)$ and $(b,v)$.
Suppose we try and write production rules:

$$
\begin{array}{lcl}
a \wedge \neg v & \mapsto & u\uparrow \\
\neg a \vee v & \mapsto & u\downarrow
\end{array}
$$

$$
\begin{array}{lcl}
b \wedge \neg u & \mapsto & v\uparrow \\
\neg b \vee u & \mapsto & v\downarrow
\end{array}
$$

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## Arbiters

To make the circuit directly implementable, we flip the sense of variables $u$ and $v$.

$$
\begin{array}{rcl}
a \wedge \_v & \mapsto & \_u\downarrow \\
\neg a \vee \neg\_v & \mapsto & \_u\uparrow
\end{array}
$$

$$
\begin{array}{rcl}
b \wedge \_u & \mapsto & \_v\downarrow \\
\neg b \vee \neg\_u & \mapsto & \_v\uparrow
\end{array}
$$

$\Rightarrow$ cross-coupled NAND gates.

What happens if both $a$ and $b$ go up at the same time?

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## Arbiters

The signals will separate eventually; however, we don't know how long it will take. It is impossible to have a circuit that decides which input switched first in bounded time.

$$\Pr[time \geq t] = Ae^{-t/\tau_0}$$

Note: the average time taken for signals to separate is bounded.

Since our circuits are asynchronous, we can wait until the signals separate.

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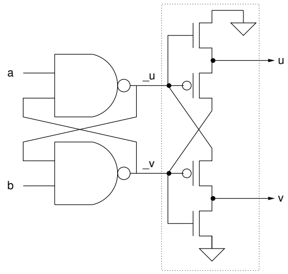
## Arbiters

The output of the cross-coupled NAND gate is connected to a filter circuit that waits for the signals to be separated by a threshold voltage.




(Note that the CMOS circuit is indeed weakly fair!)

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## Arbitration

Simple example:

$$
\begin{array}{l}
*[[\overline{A} \longrightarrow X; A \\
\;\;|\,\overline{B} \longrightarrow Y; B \\
]]
\end{array}
$$

Handshaking:

$$
\begin{array}{l}
*[[ai \longrightarrow xo\uparrow; [xi]; ao\uparrow; [\neg ai]; xo\downarrow; [\neg xi]; ao\downarrow \\
\;\;|\,bi \longrightarrow yo\uparrow; [yi]; bo\uparrow; [\neg bi]; yo\downarrow; [\neg yi]; bo\downarrow \\
]]
\end{array}
$$

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## Arbitration

Introduce new variables $u$ and $v$:

$$
\begin{array}{l}
*[[ai \longrightarrow u\uparrow; [u]; xo\uparrow; [xi]; ao\uparrow; \\
\qquad [\neg ai]; u\downarrow; [\neg u]; xo\downarrow; [\neg xi]; ao\downarrow \\
\\
\;\;|\,bi \longrightarrow v\uparrow; [v]; yo\uparrow; [yi]; bo\uparrow; \\
\qquad [\neg bi]; v\downarrow; [\neg v]; yo\downarrow; [\neg yi]; bo\downarrow \\
]]
\end{array}
$$

The idea is to introduce the output of the arbiter into the handshaking expansion. The next step is to decompose the arbiter out of the handshaking expansion.

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## Process factorization

Idea: "factor out" an arbiter!

After process factorization:

$$
\begin{array}{l}
*[[ai \longrightarrow u\uparrow; [\neg ai]; u\downarrow \\
\quad | \, bi \longrightarrow v\uparrow; [\neg bi]; v\downarrow \\
]] \\
\\
\| \\
\\
*[[u \longrightarrow xo\uparrow; [xi]; ao\uparrow; [\neg u]; xo\downarrow; [\neg xi]; ao\downarrow \\
\quad [\!] \, v \longrightarrow yo\uparrow; [yi]; bo\uparrow; [\neg v]; yo\downarrow; [\neg yi]; bo\downarrow \\
]]
\end{array}
$$



## Process factorization

Production rules:

$$
\begin{array}{rcl}
\neg bo \wedge u & \mapsto & xo\uparrow \\
xi & \mapsto & ao\uparrow \\
(bo \vee) \neg u & \mapsto & xo\downarrow \\
\neg xi & \mapsto & ao\downarrow \\
\\
\neg ao \wedge v & \mapsto & yo\uparrow \\
yi & \mapsto & bo\uparrow \\
(ao \vee) \neg v & \mapsto & yo\downarrow \\
\neg yi & \mapsto & bo\downarrow
\end{array}
$$



## Arbitration with multiplexing

CHP Program:

$$
\begin{array}{l}
*[[\overline{A} \longrightarrow S; A \\
\quad | \, \overline{B} \longrightarrow S; B \\
]]
\end{array}
$$

Decomposition:

$$
\begin{array}{l}
*[[\overline{A} \longrightarrow P; A \\
\quad | \, \overline{B} \longrightarrow Q; B \\
]] \\
\| \\
*[[\overline{P} \longrightarrow S; P \\
\quad [\!] \, \overline{Q} \longrightarrow S; Q \\
]]
\end{array}
$$



## Arbitration with multiplexing

Handshaking:

$$
\begin{array}{l}
*[[pi \longrightarrow so\uparrow; [si]; po\uparrow; [\neg pi]; so\downarrow; [\neg si]; po\downarrow \\
\quad [\!] \, qi \longrightarrow so\uparrow; [si]; qo\uparrow; [\neg qi]; so\downarrow; [\neg si]; qo\downarrow \\
]]
\end{array}
$$

Production rules:

$$
\begin{array}{rcl}
pi \vee qi & \mapsto & so\uparrow \\
\neg pi \wedge \neg qi & \mapsto & so\downarrow
\end{array}
\qquad
\begin{array}{rcl}
si \wedge qi & \mapsto & qo\uparrow \\
(\neg qi \wedge) \neg si & \mapsto & qo\downarrow
\end{array}
$$

$$
\begin{array}{rcl}
si \wedge pi & \mapsto & po\uparrow \\
(\neg pi \wedge) \neg si & \mapsto & po\downarrow
\end{array}
$$

## Negated probes

Consider the following CHP program:

$$
\begin{array}{l}
*[\,[\overline{X} \wedge \overline{S} \longrightarrow S!\mathbf{true}, X \\
\quad |\, \neg\overline{X} \wedge \overline{S} \longrightarrow S!\mathbf{false} \\
\quad ]\,]
\end{array}
$$

This program determines the current value of the probe.
$S$ determines when the probe is evaluated.

- Why a thin bar?!

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## Negated probes

Assuming the channels are passive, we get the following handshaking expansion:

$$
\begin{array}{l}
*[\,[Xi \wedge Si \longrightarrow Sto\uparrow;\ [\neg Si];\ Sto\downarrow;\ Xo\uparrow;\ [\neg Xi];\ Xo\downarrow \\
\quad |\, \neg Xi \wedge Si \longrightarrow Sfo\uparrow;\ [\neg Si];\ Sfo\downarrow \\
\quad ]\,]
\end{array}
$$

Since the CMOS implementation of a two-way arbiter is weakly fair, we can implement this HSE with the following:

$$
\begin{array}{l}
*[\,[Xi \longrightarrow [Si];\ Sto\uparrow;\ [\neg Si];\ Sto\downarrow;\ Xo\uparrow;\ [\neg Xi];\ Xo\downarrow \\
\quad |\, Si \longrightarrow Sfo\uparrow;\ [\neg Si];\ Sfo\downarrow \\
\quad ]\,]
\end{array}
$$

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## Negated probes

Introduce arbiter variables:

$$
\begin{array}{l}
*[\,[Xi \longrightarrow u\uparrow;\ [u];\ [Si];\ Sto\uparrow;\ [\neg Si];\ Sto\downarrow; \\
\qquad\qquad Xo\uparrow;\ [\neg Xi];\ u\downarrow;\ [\neg u];\ Xo\downarrow \\
\quad |\, Si \longrightarrow v\uparrow;\ [v];\ Sfo\uparrow;\ [\neg Si];\ v\downarrow;\ [\neg v];\ Sfo\downarrow \\
]\,]
\end{array}
$$

Apply process factorization:

$$
\begin{array}{l}
*[\,[u \longrightarrow [Si];\ Sto\uparrow;\ [\neg Si];\ Sto\downarrow;\ Xo\uparrow;\ [\neg u];\ Xo\downarrow \\
\quad [\!]\, v \longrightarrow Sfo\uparrow;\ [\neg v];\ Sfo\downarrow \\
\quad ]\,] \\
\| \\
\quad Arb(Xi, Si, u, v)
\end{array}
$$

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## Negated probes

Reshuffled HSE:

$$
\begin{array}{l}
*[\,[u \longrightarrow [Ei];\ Eto\uparrow;\ [\neg Ei];\ Xo\uparrow;\ Eto\downarrow;\ [\neg u];\ Xo\downarrow \\
\quad [\!]\, v \longrightarrow Efo\uparrow;\ [\neg v];\ Efo\downarrow \\
\quad ]\,]
\end{array}
$$

Production rules:

$$
\begin{array}{rcl@{\qquad\qquad}rcl}
u \wedge \_Xo \wedge Ei & \mapsto & \_Eto\downarrow & Xo & \mapsto & \_Xo\downarrow \\
\neg\_Xo & \mapsto & \_Eto\uparrow & \neg Xo & \mapsto & \_Xo\uparrow \\
\\
\_Xo \wedge v & \mapsto & Efo\downarrow & u & \mapsto & \_u\downarrow \\
\neg v & \mapsto & Efo\uparrow & \neg u & \mapsto & \_u\uparrow \\
\\
\neg\_u \wedge \neg\_Eto \wedge \neg Ei & \mapsto & Xo\uparrow \\
\_u \wedge \_Eto & \mapsto & Xo\downarrow
\end{array}
$$

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