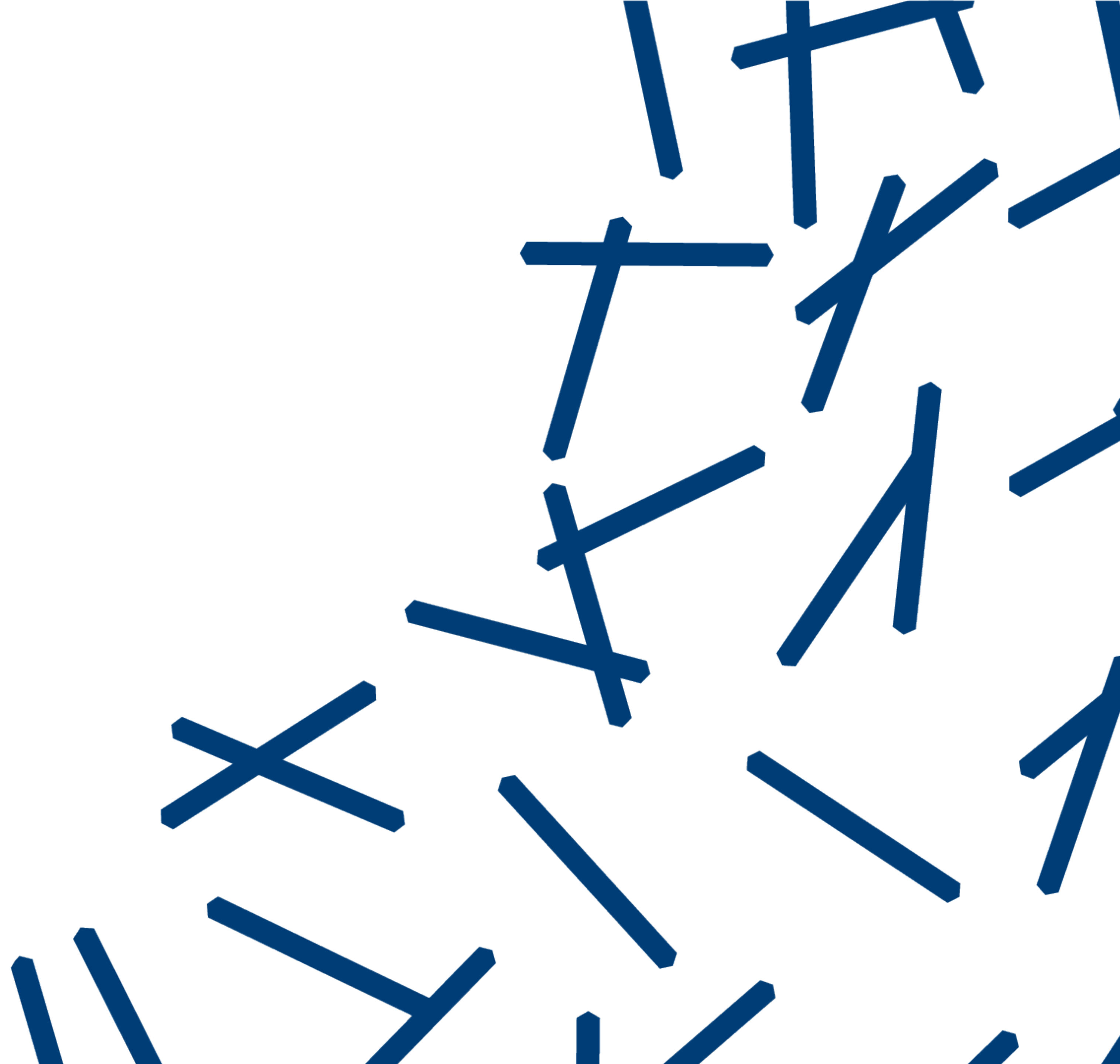


Non-determinism

Rajit Manohar



Non-deterministic selection

```
* [ [ | #A -> A?x  
    [ ] #B -> B?x  
    | ] ;  
    Z!x  
]
```

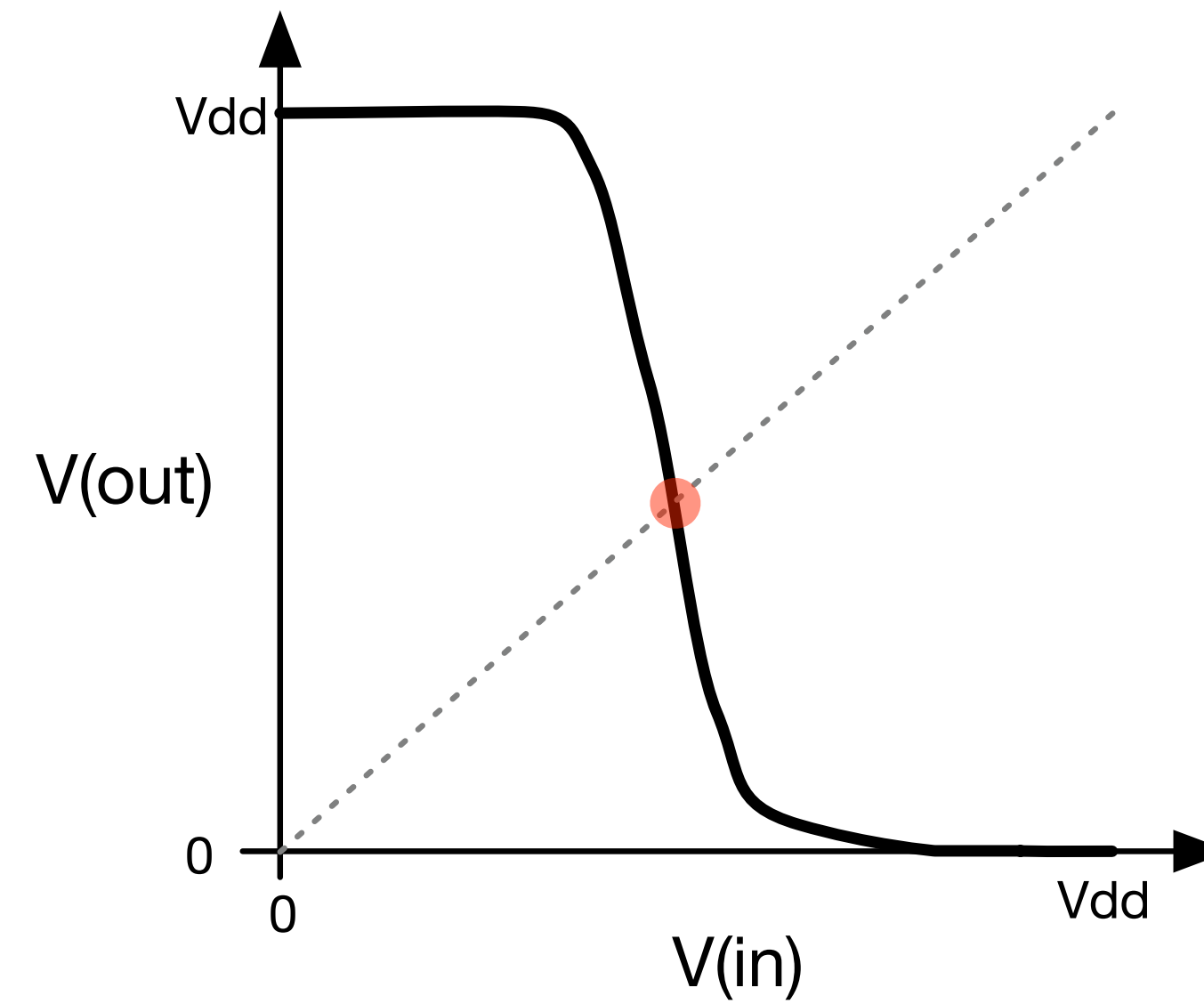
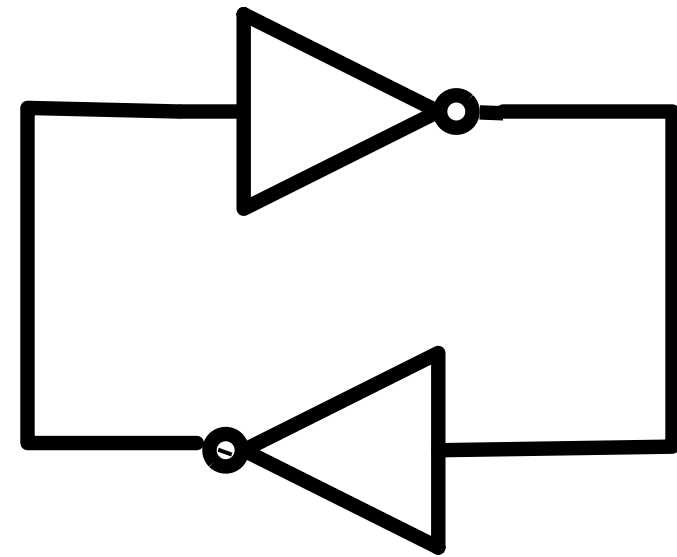
- When a communication is pending on both A and B
 - ❖ We can pick either
 - ... but we have to make a choice
 - ❖ The **arbitration** problem

Should two courses be judged equal, then the will cannot break the deadlock, all it can do is to suspend judgement until the circumstances change, and the right course of action is clear.

— Jean Buridan, c. 1340

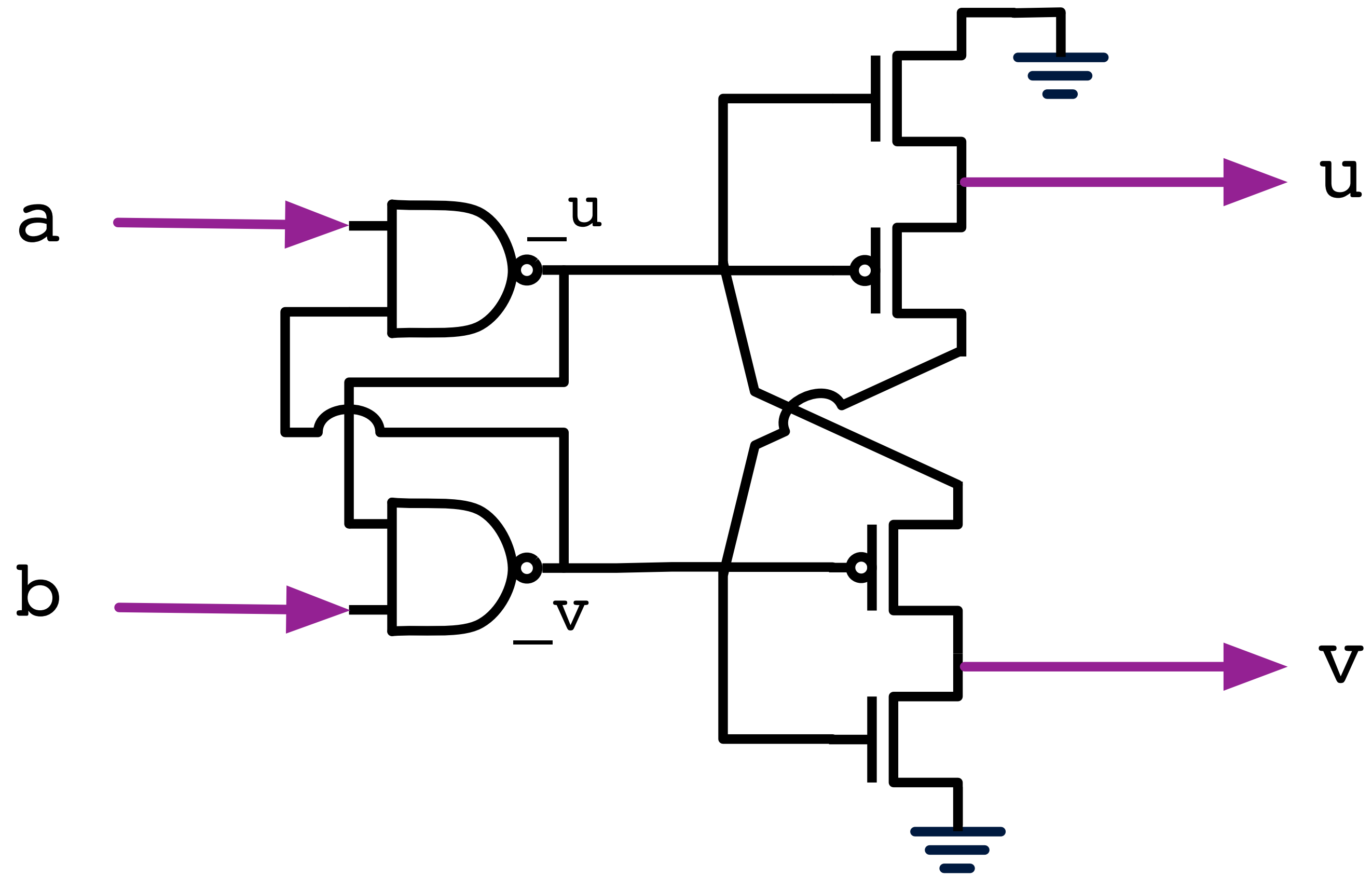
Metastable state

- Unstable equilibrium
 - ❖ Inverted pendulum
 - ❖ Balancing a pencil on its tip

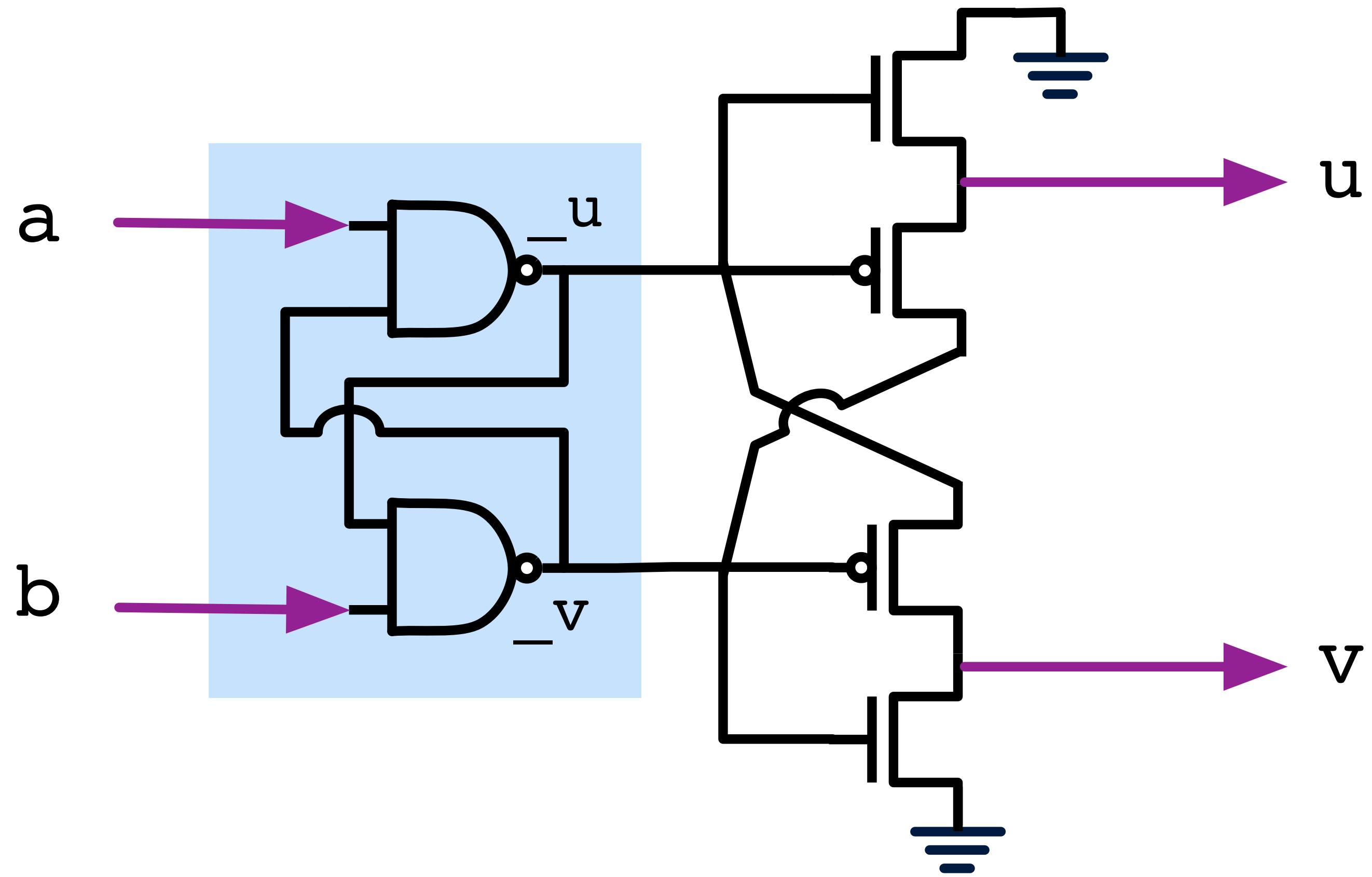


- Another example: cross-coupled inverter
 - ❖ Stable states: 0/1 and 1/0
 - ❖ Metastable state: $V(\text{out}) = V(\text{in})$

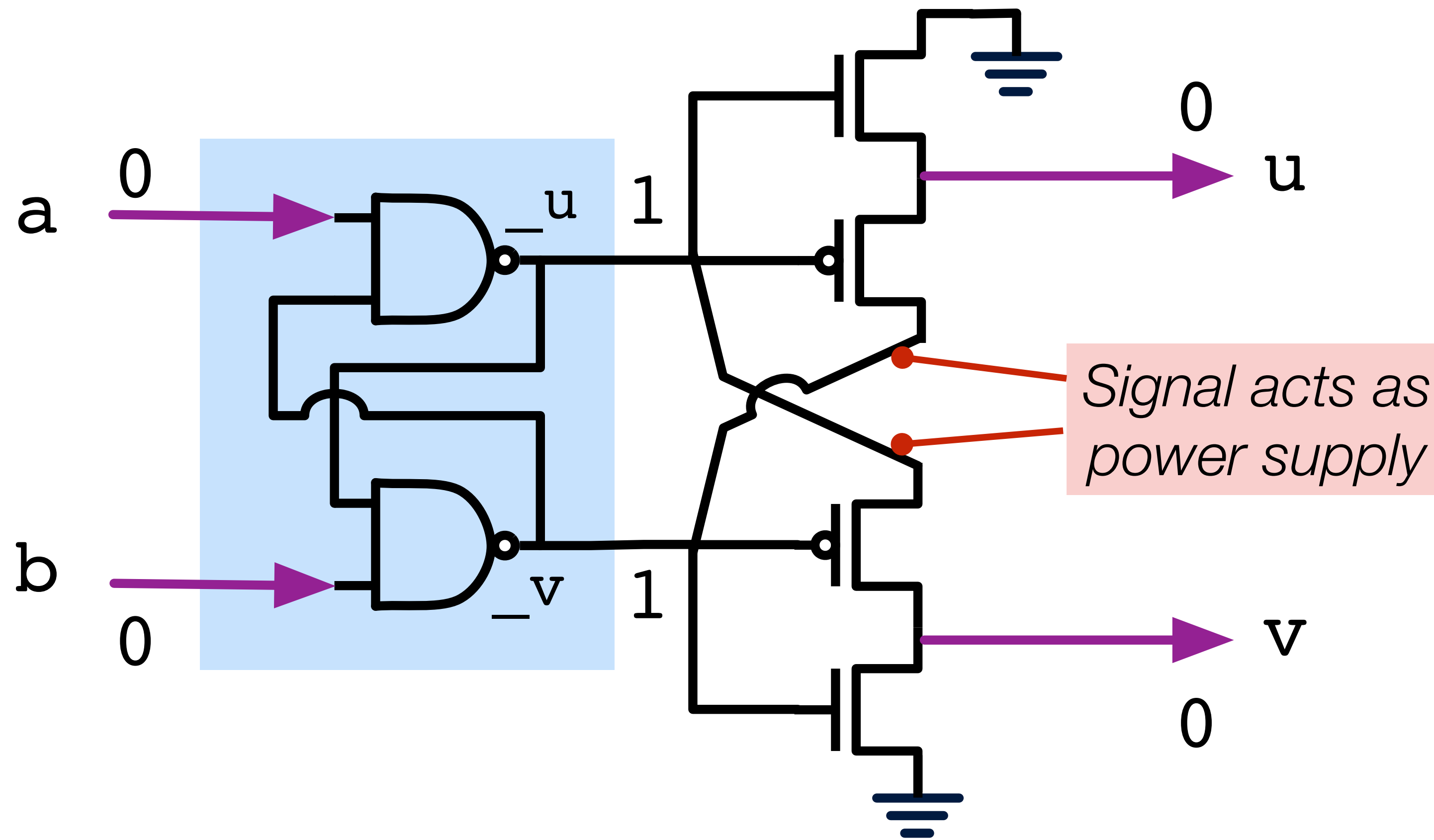
Arbiter



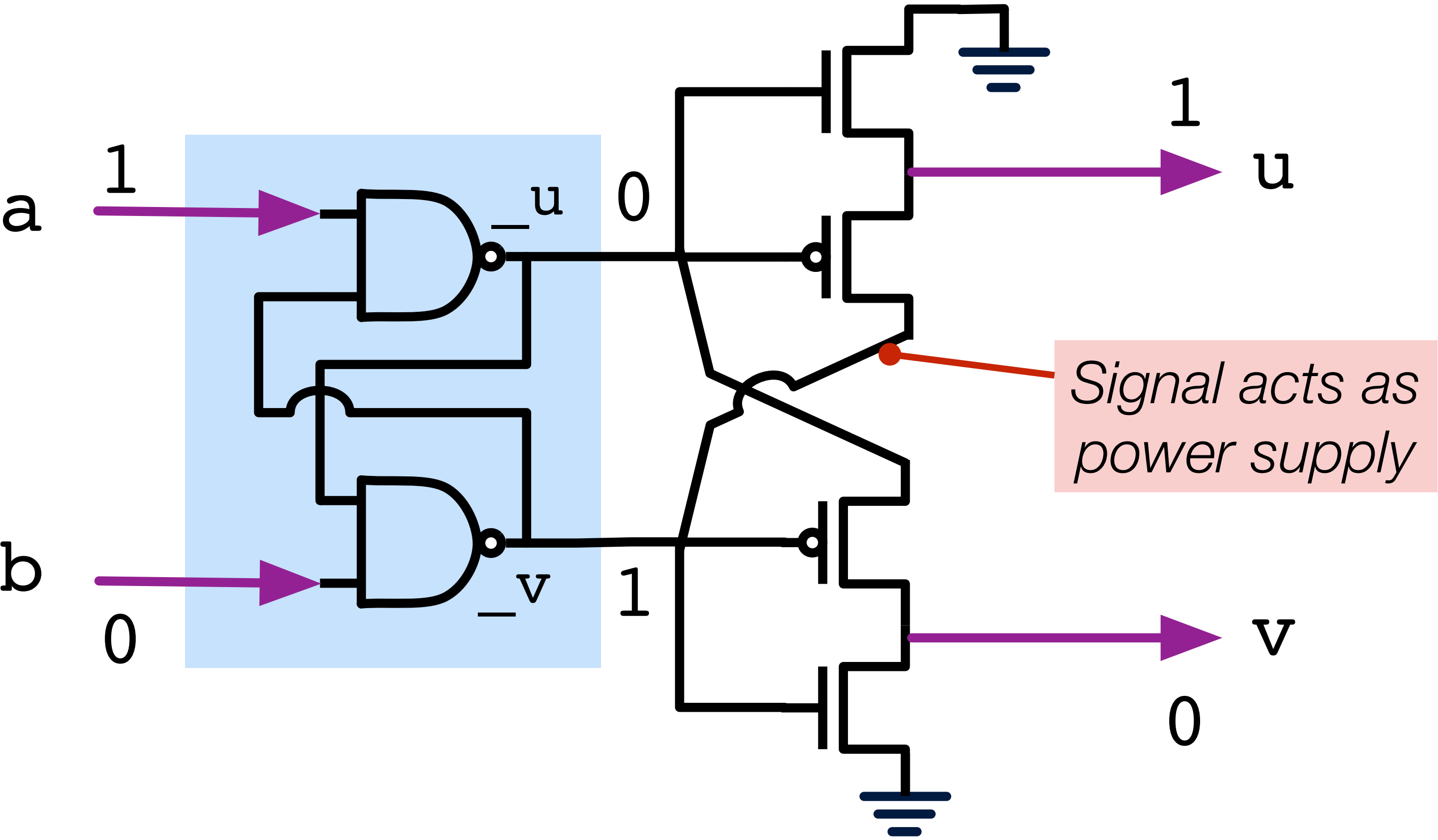
Arbiter



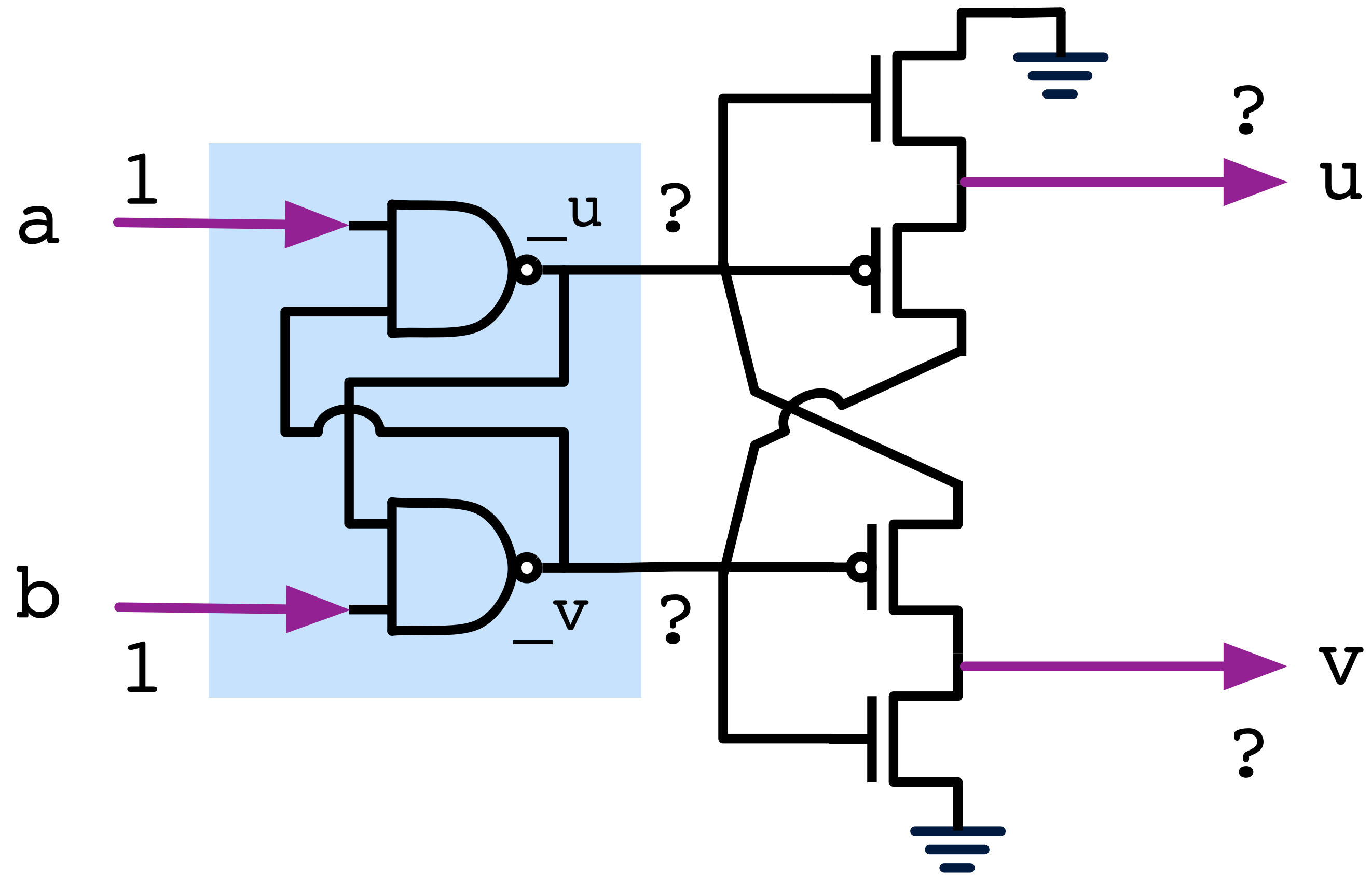
Arbiter



Arbiter

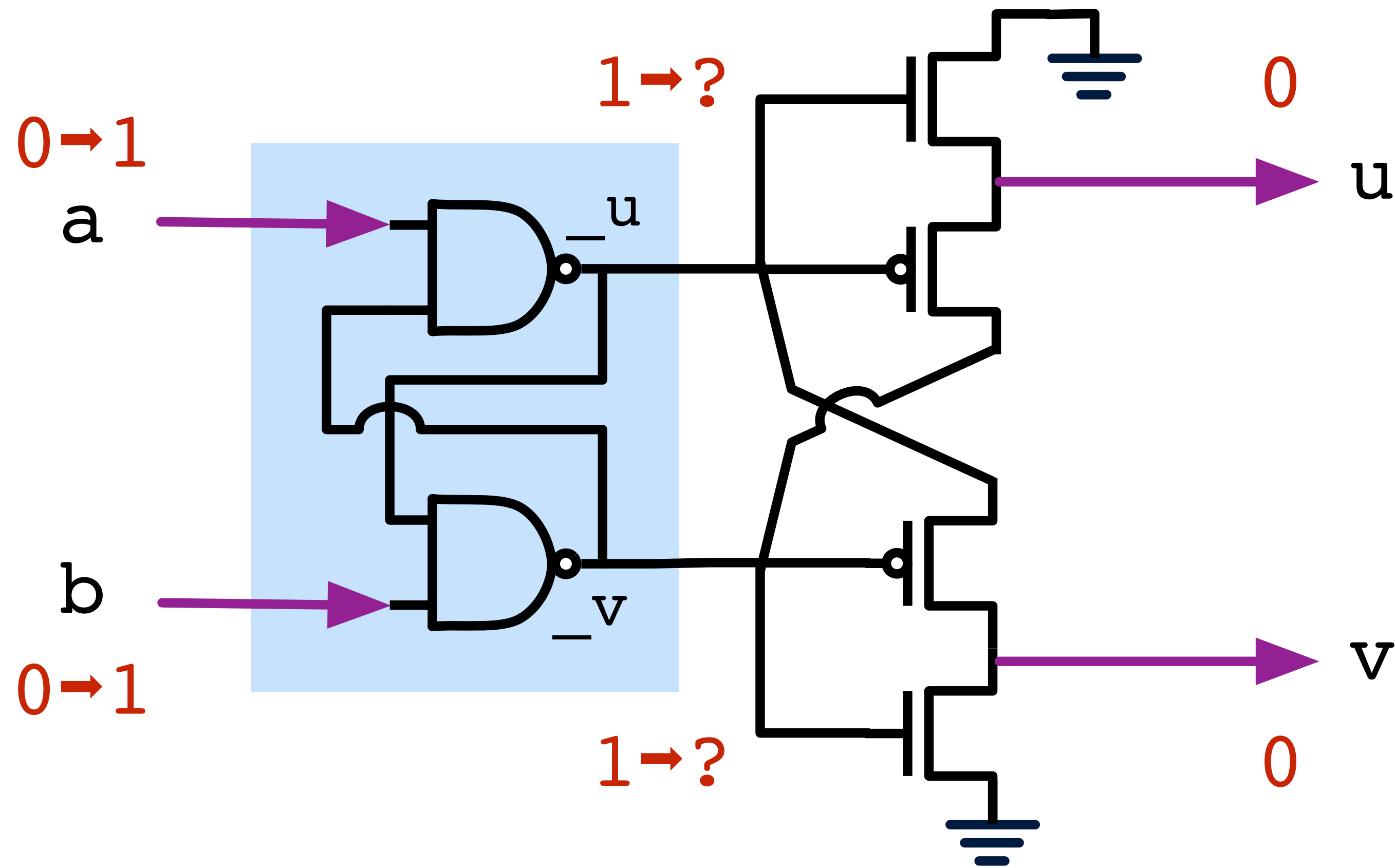


Arbiter



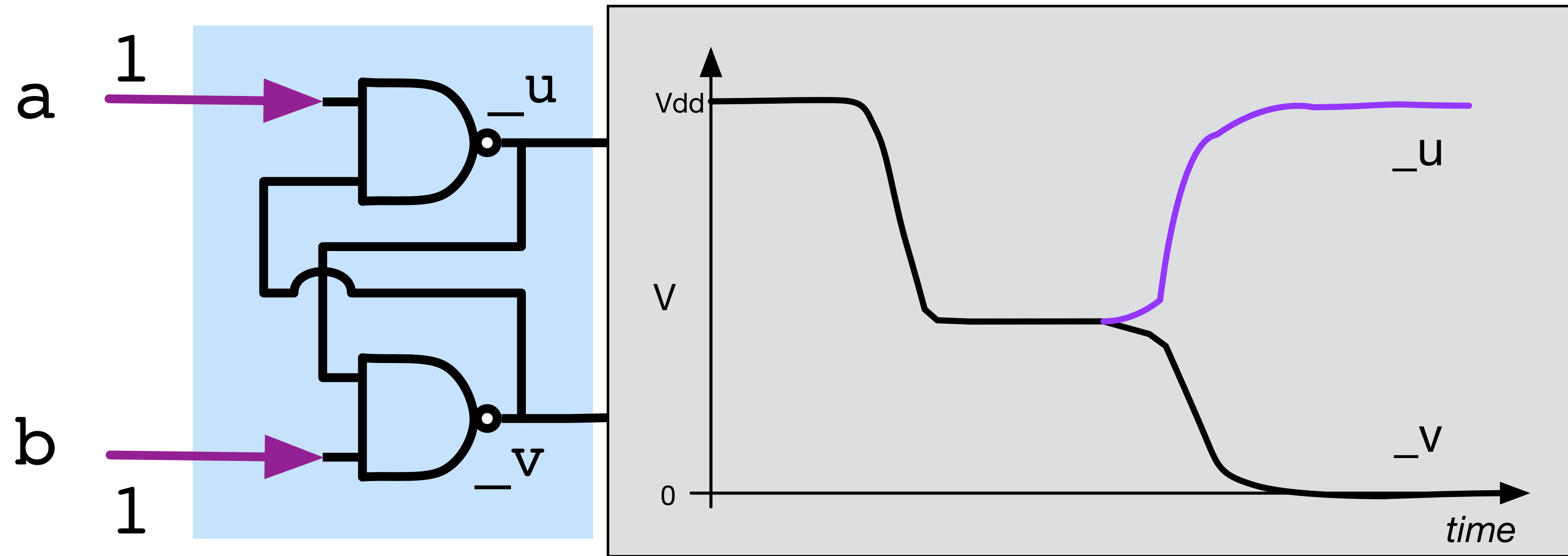
$$\Pr[\text{time} \geq t] = Ae^{-t/\tau}$$

Arbiter



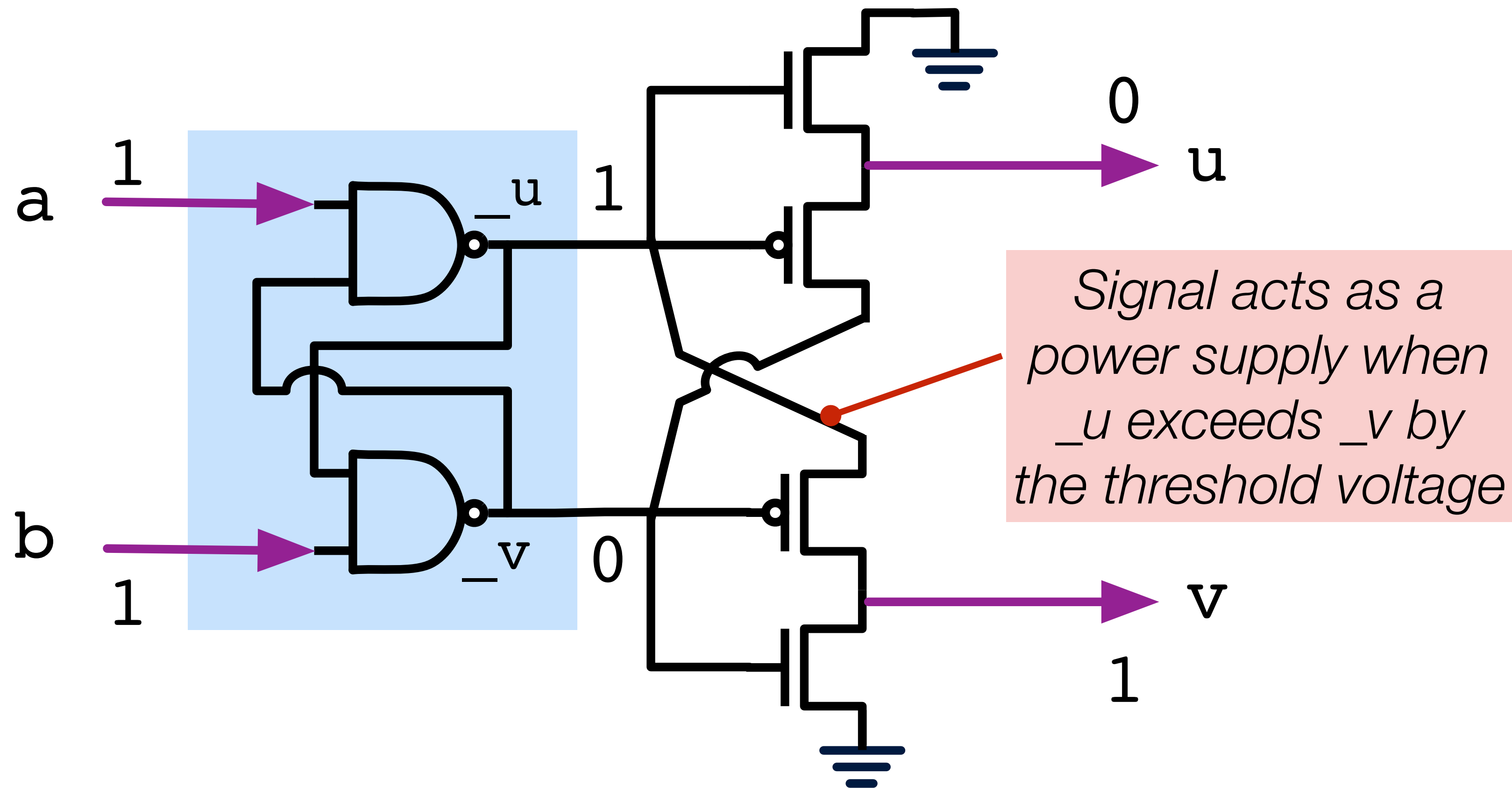
$$\Pr[\text{time} \geq t] = Ae^{-t/\tau}$$

Arbiter



$$\Pr[\text{time} \geq t] = Ae^{-t/\tau}$$

Arbiter

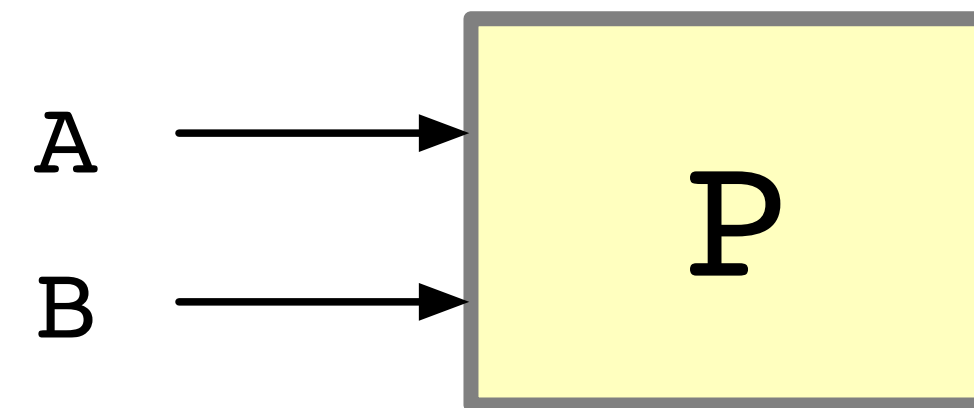


$$\Pr[\text{time} \geq t] = Ae^{-t/\tau}$$

Translating non-deterministic selections

- Basic idea

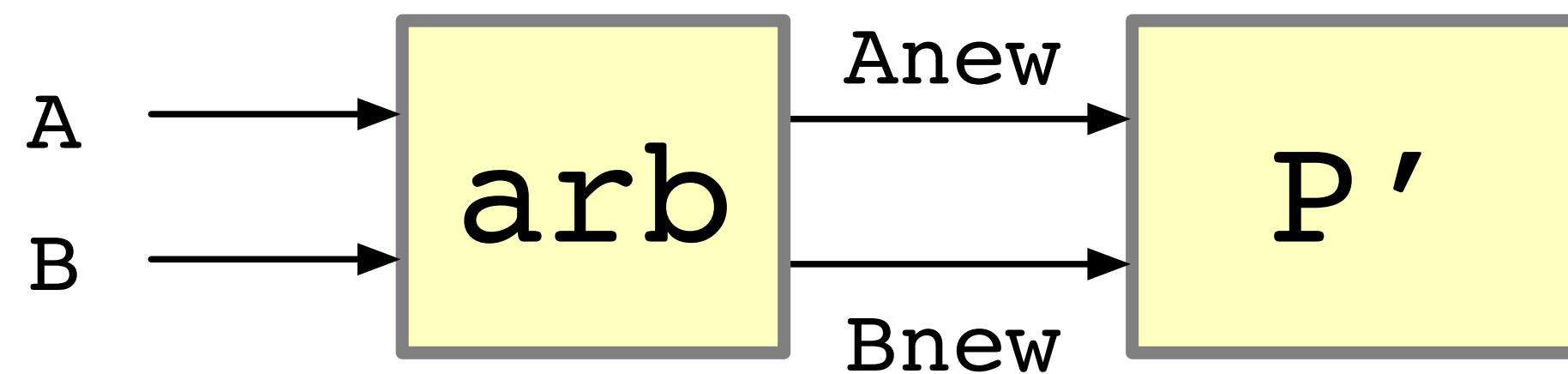
```
[ | #A -> A?x  
[ ] #B -> B?x  
| ]
```



- ❖ Factor out non-deterministic execution

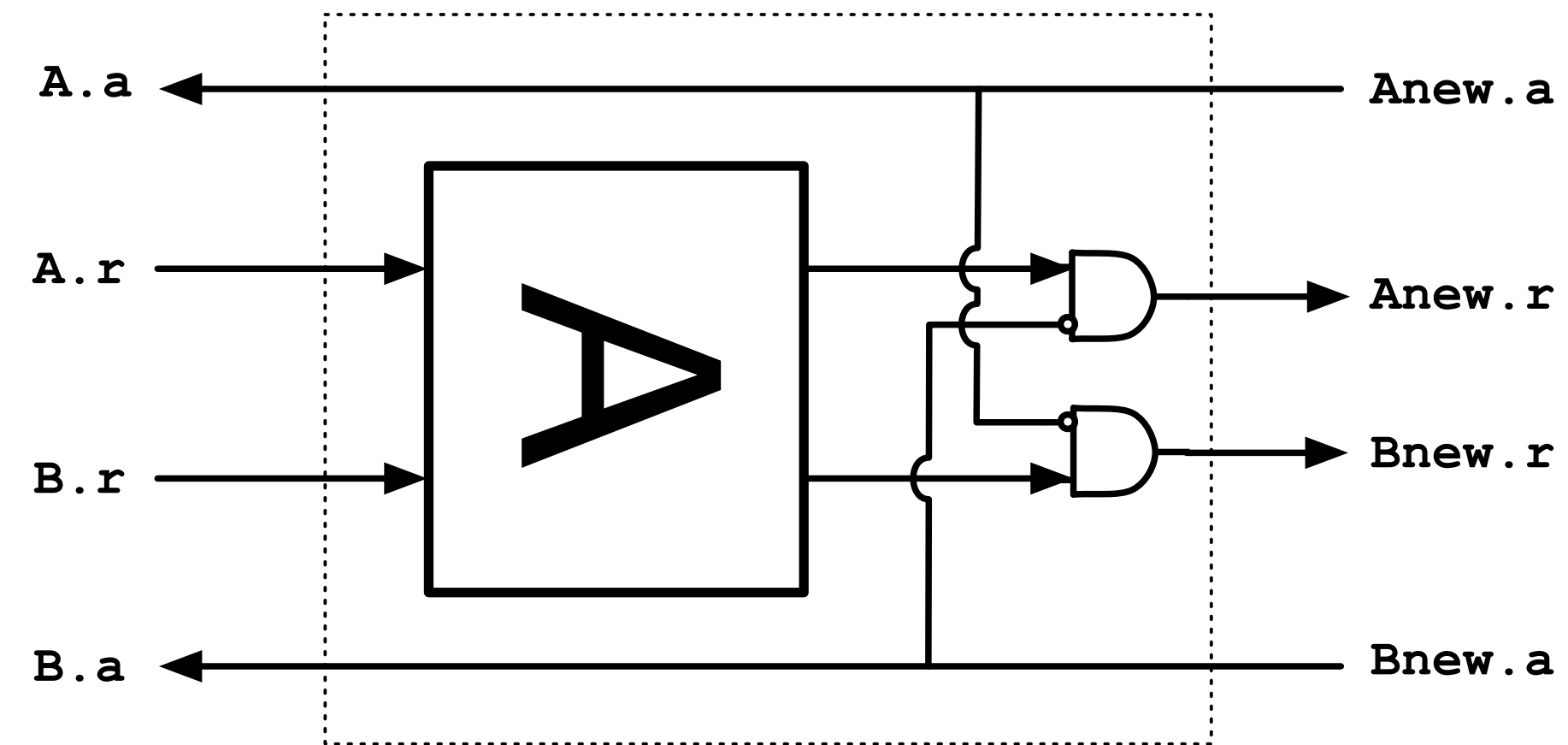
```
* [  
  [ | #A -> Anew!A;A?  
  [ ] #B -> Bnew!B;B?  
  | ]  
]
```

```
[ #Anew -> Anew?x  
[ ] #Bnew -> Bnew?x  
]
```

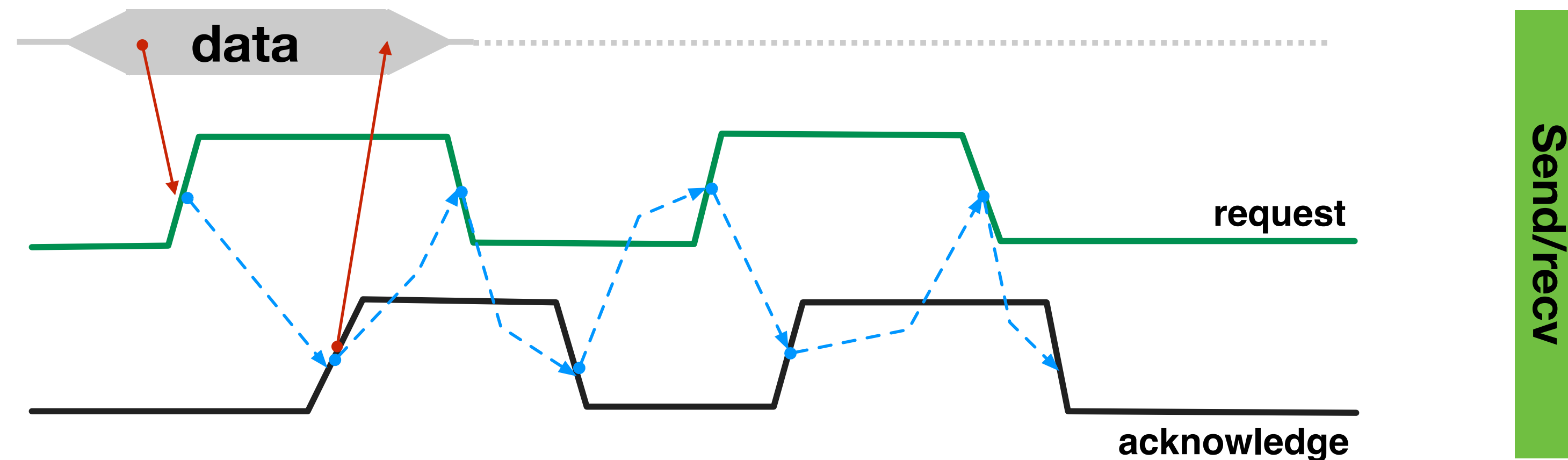


Example circuit

```
* [  
  [ | #A -> Anew!A;A?  
  [ ] #B -> Bnew!B;B?  
  | ]  
  ]
```

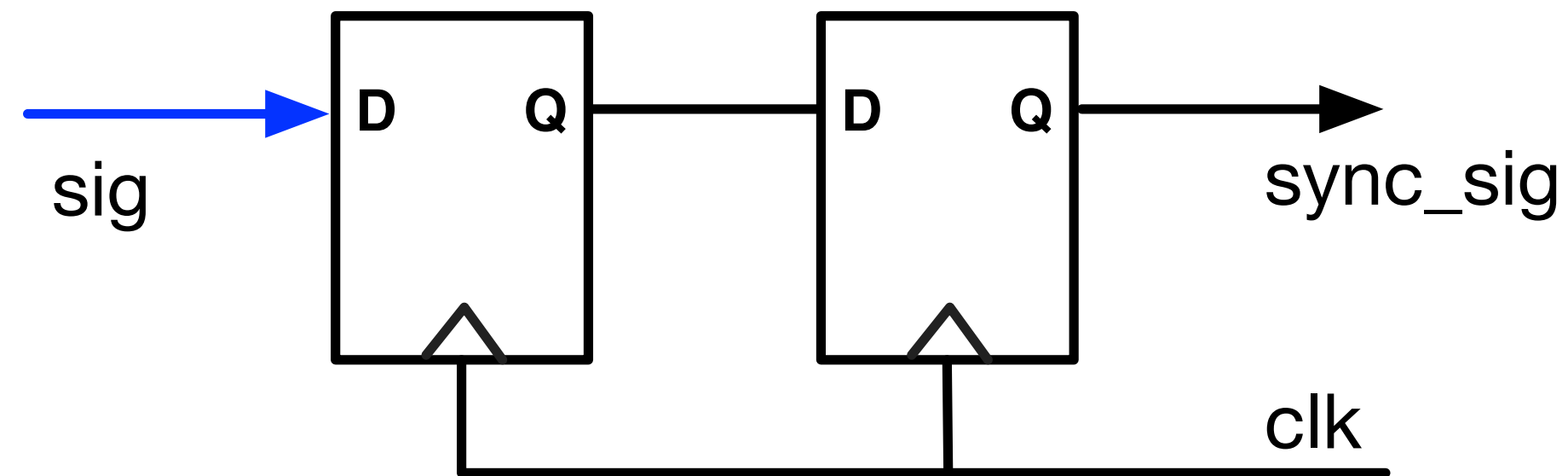


A simple (slow) interface to clocked environments

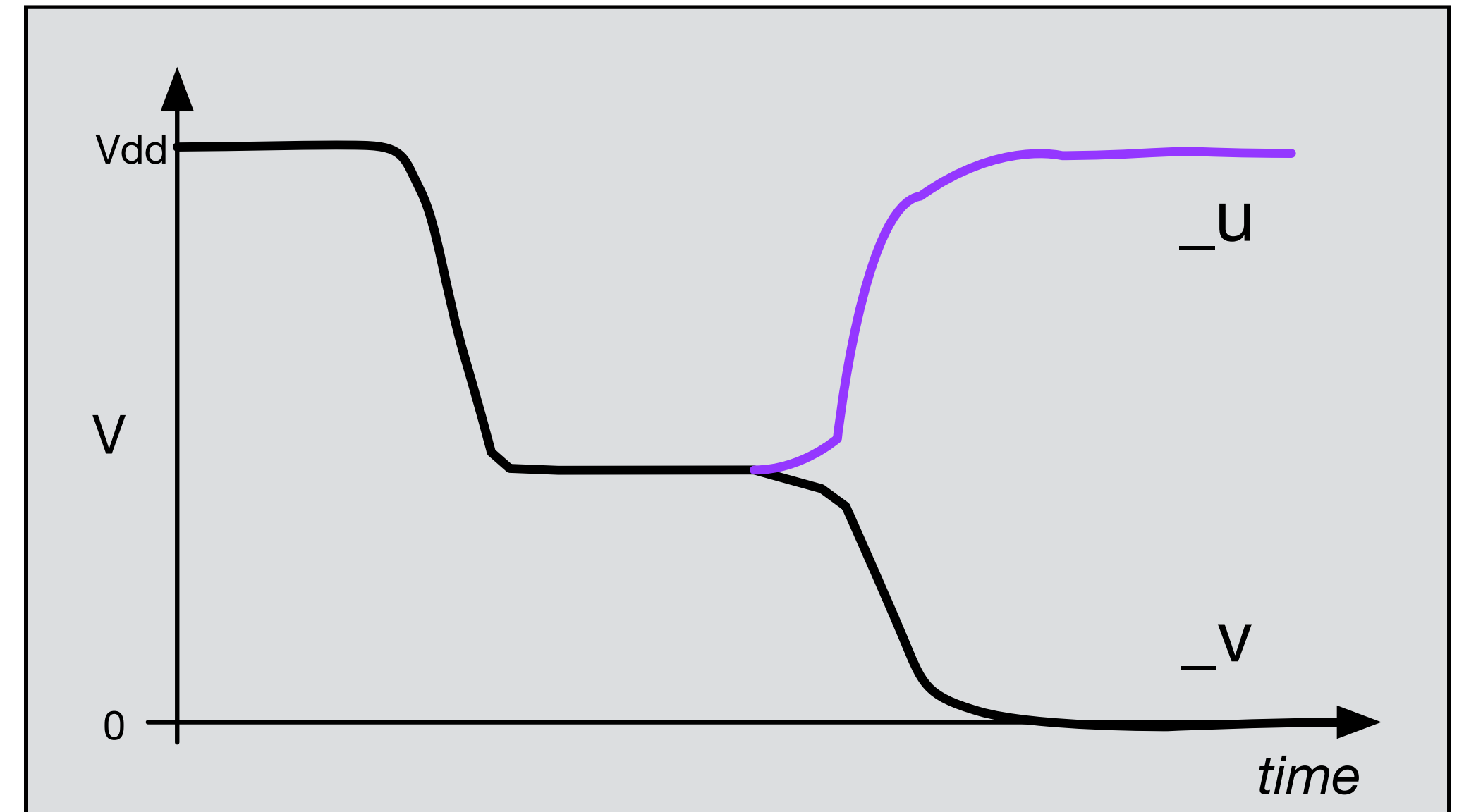


- Send to clocked environment
 - ❖ Request might change when the clock is changing
- Receive from clocked environment
 - ❖ Acknowledge might change when the clock is changing

Standard synchronizer: a sequence of flip-flops



- Send to clocked environment
 - ❖ Request might change when the clock is changing
- Receive from clocked environment
 - ❖ Acknowledge might change when the clock is changing



$$\Pr[\text{time} \geq t] = Ae^{-t/\tau}$$