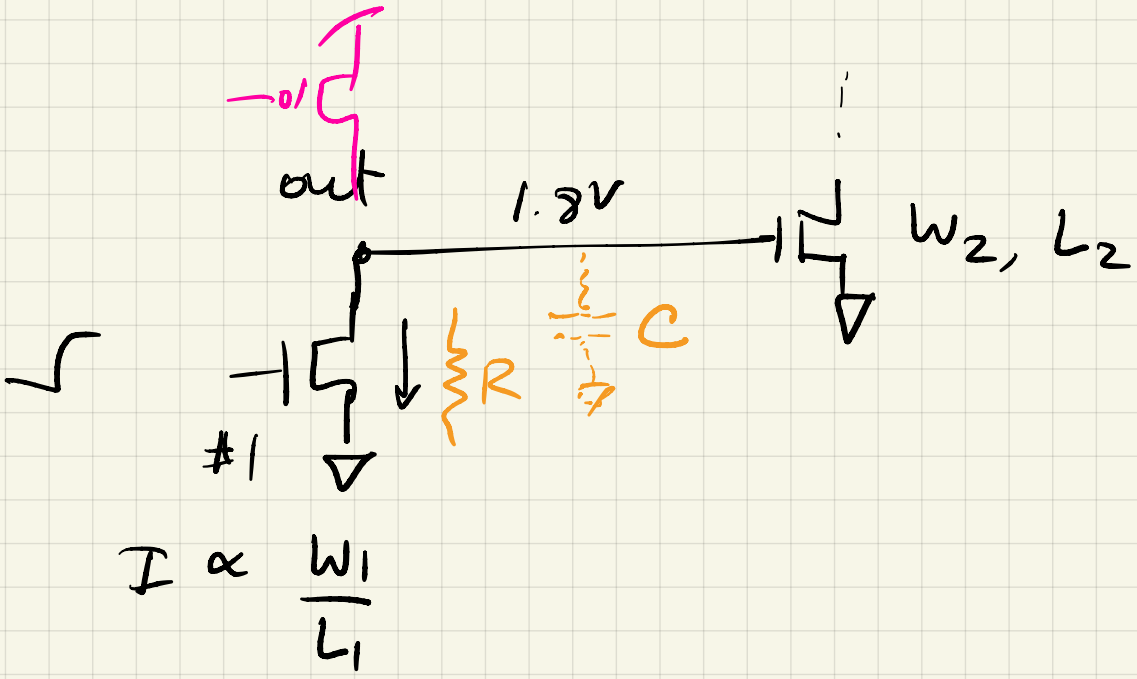
Sizing



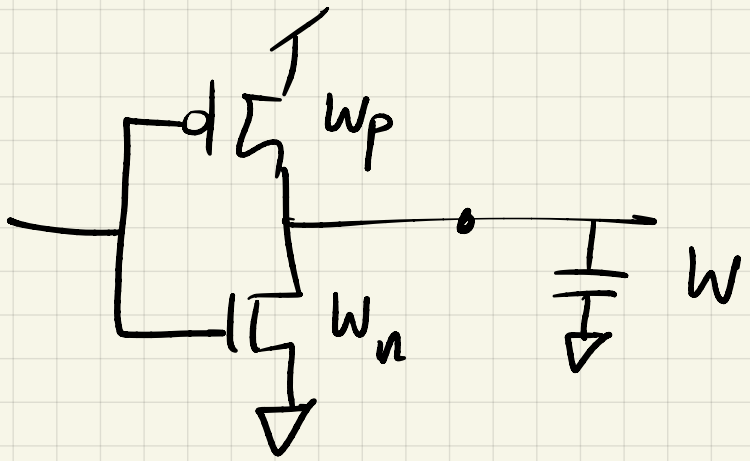
$C \propto W_2 L_2$

$R \propto \frac{L_1}{W_1}$

delay $(1 \to 0) \propto L_1 L_2 \frac{W_2}{W_1} = \tau_n \frac{W_2}{W_1}$

delay $(0 \to 1) \propto$ --- $= \tau_p \frac{W_2}{W_1}$

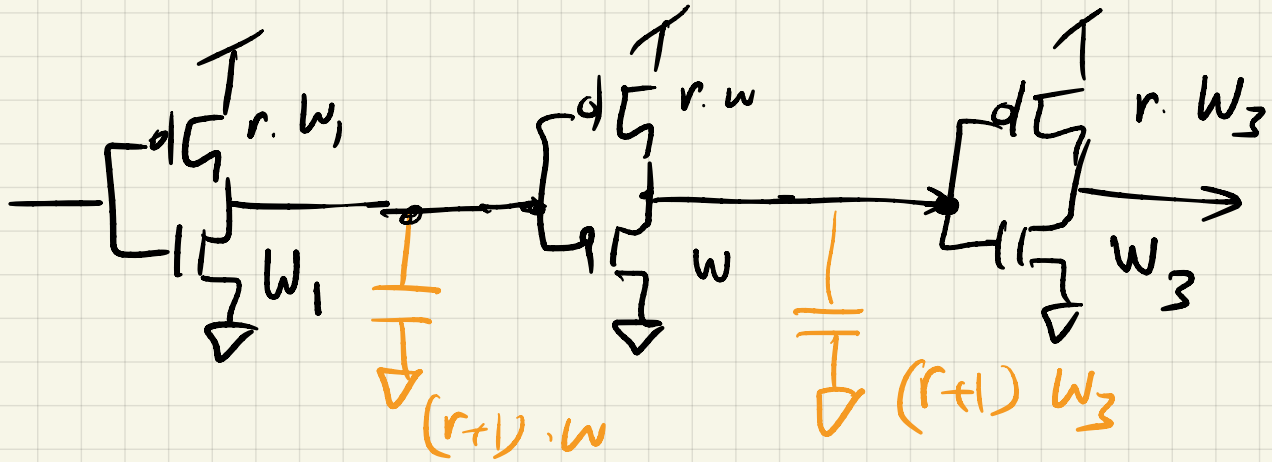
accounts for different device types.



$$d(0 \to 1) = \tau_p \cdot \frac{W}{W_p} + \Delta \qquad \tau_p \frac{W}{W_{tot} - W_n}$$

$$d(1 \to 0) = \tau_n \cdot \frac{W}{W_n} - \Delta \qquad \tau_n \frac{W}{W_n}$$

$$\frac{W_p}{W_n} = \boxed{\frac{\tau_p}{\tau_n} = r}$$



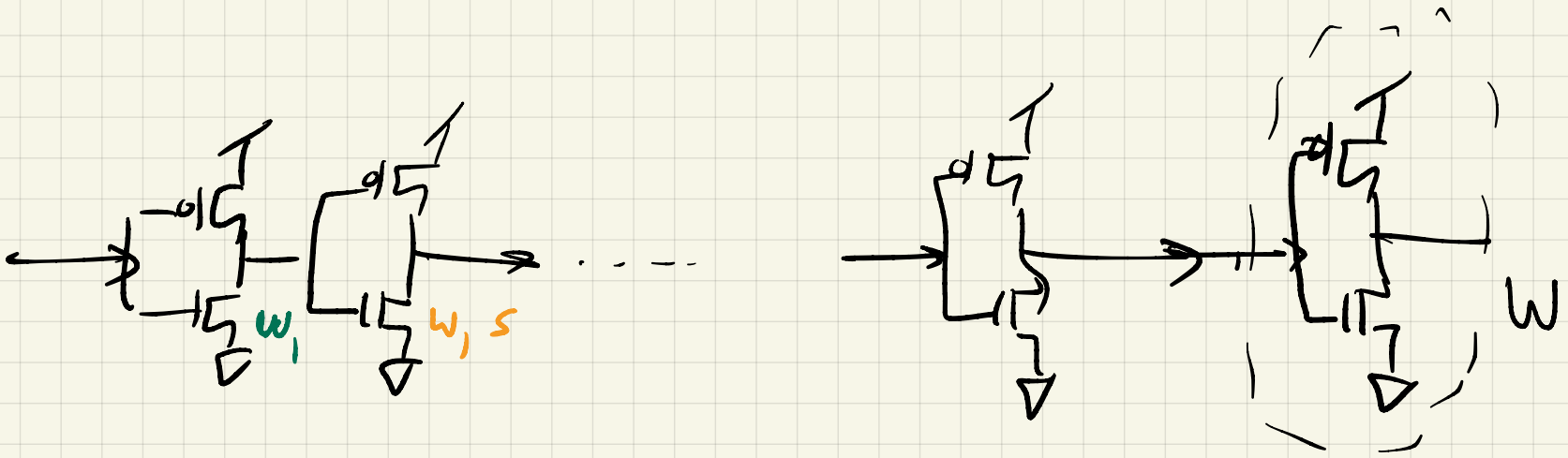
$$\text{delay} = \tau_n \cdot \frac{(r+1)w}{w_1} + \tau_n \frac{(r+1)w_3}{w}$$

$$\min_w \quad \tau_n \frac{(r+1)w}{w_1} + \tau_n \frac{(r+1)w_3}{w}$$

$$\frac{w}{w_1} + \frac{w_3}{w} \longrightarrow \frac{w_3}{w^*} = \sqrt{\frac{w_3}{w_1}}$$

$$\frac{w^*}{w_1} = \sqrt{\frac{w_3}{w_1}}$$

$$\frac{1}{w_1} - \frac{w_3}{w^2} = 0 \Rightarrow w^* = \sqrt{w_1 w_3}$$



ratio of widths:

$$a,\ as,\ as^2,\ \ldots,\ as^{n-1}$$

s = ratio of widths in geometric series

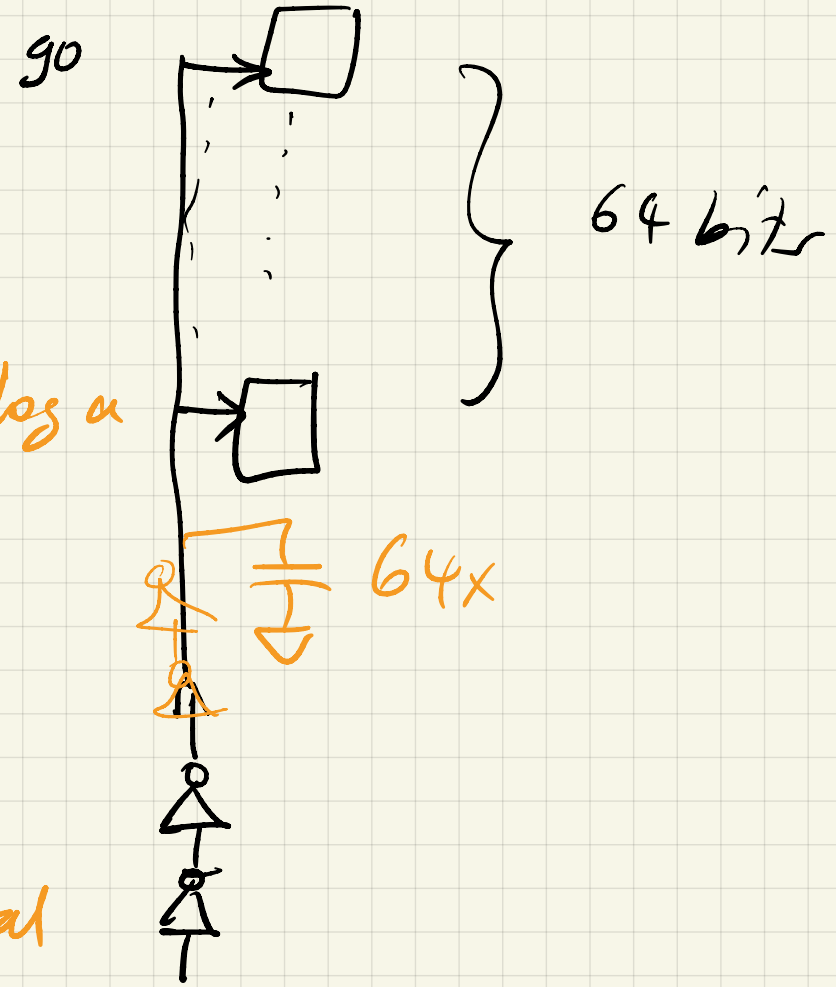
$a = w_1 \qquad as^n = W$

$$s = \sqrt[n]{\frac{W}{w_1}}$$

delay of one gate: $\tau_n \cdot \dfrac{w_1 \cdot s\,(r+1)}{w_1}$

$$= \tau_n \cdot s(r+1) = \underbrace{\tau_n (r+1)} \cdot \left(\frac{W}{w_1}\right)^{\frac{1}{n}}$$

Total delay $= \tau_n (r+1) \cdot N \cdot \left(\frac{W}{W_1}\right)^{\frac{1}{N}}$



$N \cdot a^{\frac{1}{N}}$

$\frac{d}{dN} = a^{\frac{1}{N}} - \frac{1}{N^2} \cdot a^{\frac{1}{N}} \cdot N \log a$

$= 0$

$\Rightarrow N = \log a$

ln natural log.